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RELIABILITY ANALYSIS OF TWO DISSIMILAR PARALLEL UNIT OIL DELIVERING SYSTEM WITH PREVENTIVE MAINTENANCE AND CORRELATED LIFE TIME

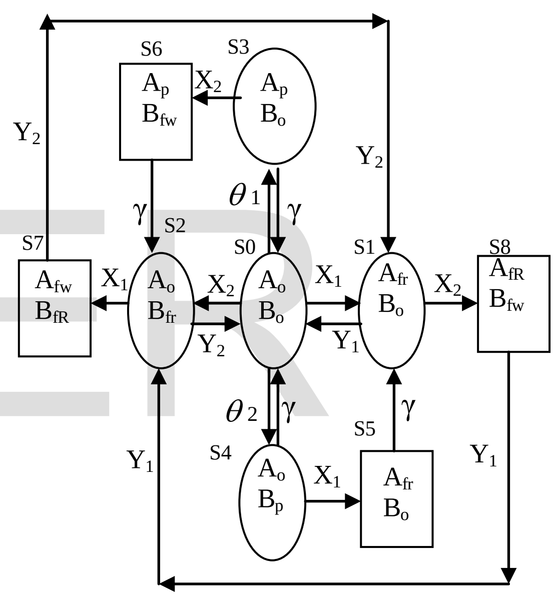
Mohit Kakkar,Ashok Chitkara,Jasdev Bhatti

ABSTRACT In this paper we study a two dissimilar parallel unit oil delivering system with preventive maintenance of both units with one repairman under the assumption that each unit is as good as new after the preventive maintenance and repair. System will fail when both the units fails totally. Failure and Repair times of each unit are assumed to be correlated. Using regenerative point technique various reliability characteristics are obtained which are useful to system designers and industrial managers. MTSF and the profit functions are studied graphically

KEYWORDS Regenerative point technique, Transition Probabilities, MTSF, Availability, Busy Period, Profit Function..

1. INTRODUCTION

The reliability analysis of a variety of system models with identical/non-identical units and different repair and failure policies exist in the literature on reliability theory. A two dissimilar parallel unit systems have widely studied in literature of reliability theory and repair maintenance is one of the most important measures for increasing the reliability of the system. Many authors have studied various system models under different repair policies [3-4] they have assumed that the failure and repair times are uncorrelated random variable. During our visit in an oil refinery we have seen that in an oil delivering system after repair, a unit recovers its function perfectly. An operative unit (A or B) goes to a specified time t and it is free from failure in that interval, the unit undergoes inspection as the PM. We also assume that the switching over times from the repair to the PM and from the PM to the operative state are assumed to be instantaneous. Taking this fact into consideration in this paper we investigate a two dissimilar parallel unit oil delivering system model in an oil refinery with failure and repair of unit as correlated random variables having their joint distribution as bivariate exponential .



TRANSITION DIAGRAM

2.0 SYSTEM DESCRIPTION

System consists of two non-identical units both are operative (A and B). There is single repair facility. Every unit work as good as new after repair. Repair equipment is perfect during the repair of any failed unit. System is down when both the units are non operative. Failure and Repair times of each unit are assumed to be correlated. The joint distribution of failure and repair times for each unit is taken to be bivariate exponential having density function.

3.0 NOTATIONS

A_0 :	Unit A is in operative mode
B_0:	Unit B is in operative mode
A_{fr} :	Unit A is in failure mode
B_{fr} :	Unit B is in failure mode
A_P :	Unit A is in PM mode
B_P :	Unit B is in PM mode
θ_1 / θ_2 **:**	Constant rate for taking a unit into PM (for both units A and B respectively)

γ_1 / γ_2: Constant rate of ending of PM (for both units A and B respectively)

A$_{Fw}$: Unit A in failure mode but in waiting for repairman

B$_{Fw}$: Unit B in failure mode but in waiting for repairman

X$_i$(i=1,2) Random variables representing the failure times of A and B unit respectively for i=1,2

Y$_i$(i=1,2) Random variables representing the repair times of A and B unit respectively for i=1,2

f$_i$**(x,y):** Joint pdf of (x$_i$,y$_i$);i=1,2

$$= \alpha_i \beta_i (1-r_i) e^{-\alpha_i x - \beta_i y} I_0 (2\sqrt{(\alpha_i \beta_i r_i xy)}); X, Y, \alpha_i, \beta_i > 0;$$
$$0 \le r_i < 1,$$
$$where I_0 (2\sqrt{\alpha_i \beta_i r_i xy}) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i xy)^j}{(j!)^2}$$

k$_i$**(Y/X):** Conditional pdf of Y$_i$ given X$_i$=x is given by

$$= \beta_i e^{-\alpha_i r_i x - \beta_i y} I_0 (2\sqrt{(\alpha_i \beta_i r_i xy)})$$

g$_i$**(.):** Marginal pdf of X$_i$= $\alpha_i (1-r_i) e^{-\alpha_i (1-r_i) x}$

h$_i$**(.):** Marginal pdf of Y$_i$= $\beta_i (1-r_i) e^{-\beta_i (1-r_i) y}$

$q_{ij}(.), Q_{ij}(.)$ pdf &cdf of transition time from regenerative states pdf &cdf of transition time from regenerative state S$_i$ to S$_j$.

μ_i : Mean sojourn time in state S$_i$.

$\oplus$: Symbol of ordinary Convolution

$$A(t) \oplus B(t) = \int_0^t A(t-u) B(u) du$$

$\otimes$: symbol of stieltjes convolution

$$A(t) \otimes B(t) = \int_0^t A(t-u) dB(u)$$

4.0 TRANSITIONS PROBABILITIES

$$P_{01} = \frac{\alpha_1 (1-r_1)}{\theta_1 + \theta_2 + \phi_1} \qquad P_{20} = \frac{\beta_2 (1-r_2)}{\beta_2 (1-r_2) + \alpha_1 (1-r_1)}$$

$$P_{02} = \frac{\alpha_2 (1-r_2)}{\theta_1 + \theta_2 + \phi_1} \qquad P_{21.7} = \frac{\alpha_1 (1-r_1)}{\beta_2 (1-r_2) + \alpha_1 (1-r_1)}$$

$$P_{03} = \frac{\theta_1}{\theta_1 + \theta_2 + \phi_1} \qquad P_{30} = \frac{\gamma_1}{\gamma_1 + \alpha_2 (1-r_2)}$$

$$P_{04} = \frac{\theta_2}{\theta_1 + \theta_2 + \phi_1} \qquad P_{32.6} = \frac{\alpha_2 (1-r_2)}{\gamma_1 + \alpha_2 (1-r_2)}$$

$$P_{10} = \frac{\beta_1 (1-r_1)}{\beta_1 (1-r_1) + \alpha_2 (1-r_2)} \qquad P_{40} = \frac{\gamma_2}{\gamma_2 + \alpha_1 (1-r_1)}$$

$$P_{12.8} = \frac{\alpha_2 (1-r_2)}{\beta_1 (1-r_1) + \alpha_2 (1-r_2)} \qquad P_{41.5} = \frac{\alpha_1 (1-r_1)}{\gamma_2 + \alpha_1 (1-r_1)}$$

Here we can see that:

$$P_{01} + P_{02} + P_{03} + P_{04} = 1 \qquad P_{20} + P_{21.7} = 1$$
$$P_{10} + P_{12.8} = 1 \qquad P_{30} + P_{32.6} = 1$$
$$P_{40} + P_{41.5} = 1 \tag{4.1-4.17}$$

5.0 ANALYSIS OF CHARACTERISTICS

5.1 Mean time to system failure

To determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

$$\phi_0 = Q_{01} \otimes \phi_1(t) + Q_{02} \otimes \phi_2(t) + Q_{03} \otimes \phi_3(t) + Q_{04} \otimes \phi_4(t)$$
$$\phi_1 = Q_{18} + Q_{10} \otimes \phi_0(t)$$
$$\phi_2 = Q_{27} + Q_{20} \otimes \phi_0(t)$$
$$\phi_3 = Q_{36} + Q_{30} \otimes \phi_0(t)$$
$$\phi_4 = Q_{45} + Q_{40} \otimes \phi_0(t) \tag{5.1-5.5}$$

Taking Laplace Stieltjes Transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)} \tag{5.6}$$

Where

$$MTSF = \frac{N_1}{D_1} \tag{5.7}$$

$$N_1 = \mu_0 + \mu_1 P_{01} + \mu_2 P_{02} + \mu_3 P_{03} + \mu_4 P_{04}$$
$$D_1 = 1 - P_{10} P_{01} - P_{20} P_{02} - P_{30} P_{03} - P_{40} P_{04}$$

(5.8-5.9)

5.2 AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at t=0.using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations

$$\begin{aligned}
&A_0 = M_0(t) + q_{01} \oplus A_1(t) + q_{02} \oplus A_2(t) \\
&+ q_{03} \oplus A_3(t) + q_{04} \oplus A_4(t) \\
&A_1 = M_1(t) + q_{10} \oplus A_0(t) + q_{12.8} \oplus A_2(t) \\
&A_2 = M_2(t) + q_{20} \oplus A_0(t) + q_{21.7} \oplus A_1(t) \\
&A_3 = M_3(t) + q_{30} \oplus A_0(t) + q_{32.6} \oplus A_2(t) \\
&A_4 = M_4(t) + q_{40} \oplus A_0(t) + q_{41.5} \oplus A_1(t)
\end{aligned}$$

(5.10-5.14)

Taking laplace transform of the equations and solving them for $A_0^*(s)$,we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

(5.11)

In the steady state

$$\begin{aligned}
&N_2 = P_{04}[\mu_4(1 - P_{12.8}P_{21.7}) + P_{41.5}(\mu_1 + \mu_2 P_{12.8})] + \\
&P_{03}[\mu_3(1 - P_{12.8}P_{21.7}) + P_{32.6}(\mu_2 + \mu_1 P_{21.7})] + \\
&\mu_0(1 - P_{12.8}P_{21.7}) + P_{01}(\mu_1 + \mu_2 P_{12.8}) + \\
&P_{02}(\mu_2 + \mu_1 P_{21.7}) \\
&D_2 = (P_{10} + P_{20} - P_{10}P_{20})[\mu_0 + \mu_3 P_{03} + \mu_4 P_{04}] \\
&+ (\mu_1 + \mu_2)[1 - P_{04}P_{40} - P_{03}P_{30}] \\
&+ \mu_1(P_{03}P_{20}P_{30} - P_{02}P_{20}) \\
&+ \mu_2(P_{04}P_{40}P_{10} - P_{01}P_{10})
\end{aligned}$$

(5.12-5.13)

5.3 BUSY PERIOD ANALYSIS

Let B_i (t) be the probability that the repairman is busy at instant t, given that the system entered regenerative state I at t=0. By probabilistic arguments we have the following recursive relations for Bi (t)

$$\begin{aligned}
&B_0 = q_{01} \oplus B_1(t) + q_{02} \oplus B_2(t) + q_{03} \oplus B_3(t) \\
&+ q_{04} \oplus B_4(t) \\
&B_1 = W_1 + q_{10} \oplus B_0(t) + q_{12.8} \oplus B_2(t) \\
&B_2 = W_2 + q_{20} \oplus B_0(t) + q_{21.7} \oplus B_1(t) \\
&B_3 = W_3 + q_{30} \oplus B_0(t) + q_{32.6} \oplus B_2(t) \\
&B_4 = W_4 + q_{40} \oplus B_0(t) + q_{41.5} \oplus B_1(t)
\end{aligned}$$

(5.14-5.18)

Taking LaplaceTansform of the equations of busy period analysis and solving them for $B_0^*(s)$,we get

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)}$$

(5.19)

In the steady state

$$B_0 = \lim_{s \to 0} sB^*{}_0(s) = \frac{N_3}{D_2}$$

(5.20)

$$\text{Busy period} = \frac{N_3}{D_2}$$

$$\begin{aligned}
&N_3 = P_{04}[\mu_4 + P_{41.5}\mu_1 + \mu_2 P_{12.8}P_{41.5} - P_{12.8}P_{21.7})] \\
&+ P_{03}[\mu_3(1 - P_{12.8}P_{21.7}) + P_{32.6}\mu_2 + \mu_1 P_{32.6}P_{21.7})] \\
&+ P_{01}(\mu_1 + \mu_2 P_{12.8}) + P_{02}(\mu_2 + \mu_1 P_{21.7})
\end{aligned}$$

D_2 already specified (5.21)

5.4 NUMBER OF VISITS BY THE REPAIRMAN

We defined as the expected number of visits by the repairman in (0,t],given that the system initially starts from regenerative state S_i, by probabilistic arguments we have the following recursive relations for $V_i(t)$

$$\begin{aligned}
&V_0 = q_{01} \otimes (1 + V_1(t)) + q_{02} \otimes (1 + V_2(t)) \\
&+ q_{03} \otimes (1 + V_3(t)) + q_{04} \otimes (1 + V_4(t)) \\
&V_1 = q_{10} \otimes V_0(t) + q_{12.8} \otimes V_2(t) \\
&V_2 = q_{20} \otimes V_0(t) + q_{21.7} \otimes V_1(t) \\
&V_3 = q_{30} \otimes V_0(t) + q_{32.6} \otimes V_2(t) \\
&V_4 = q_{40} \otimes V_0(t) + q_{41.5} \otimes V_1(t)
\end{aligned}$$

(5.22-5.26)

Taking Laplace Stieltjes Transform of the equations of expected number of visits
And solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_4}{D_2} \qquad (5.27)$$

In steady state

$$V_0 = \lim_{s\to 0}(sV^*{}_0(s)) = \frac{N_4}{D_2} \quad (5.28)$$

Where

No. of visit by the repairman $= \dfrac{N_4}{D_2}$

$N_3 = (1 - P_{12.8}P_{21.7})$

D_2 already specified (5.29)

6.0 PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2V_0 \quad (6.1)$$

Where

C_0=revenue/unit uptime of the system

C_1=cost/unit time for which repairman is busy

C_2=cost/visit for the repairman

7.0 CONCLUSION

For a more clear view of the system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and profit function in figure-1 and figure-2 w.r.t the failure parameter (α) of unit A for three different values of correlation coefficient (r_{11} =0.25, r_{12} =0.50, r_{13} =0.75), between X_1 and Y_1 and two different values of repair parameter (β) of unit A while the other parameters are kept fixed as

$$\alpha_2 = .005, \beta_1 = .03, \beta_2 = .02$$

$$C_0 = 500, C_1 = 300, C_2 = 50$$

$$\theta_1 = .004, \theta_2 = .008, \gamma_1 = .005, \gamma_2 = .001$$

From the figure-1 it is observed that MTSF decreases as failure rate increases irrespective of other parameters. the curves also indicates that for the same value of failure rate, MTSF is higher for higher values of correlation coefficient(r), so here we conclude that the high value of correlation coefficient(r) between failure and repair tends to increase the expected life time of the system. From the figure-2 it is clear that profit decreases linearly as failure rate increases. Also for the fixed value of failure rate, the profit is higher for high correlation (r).

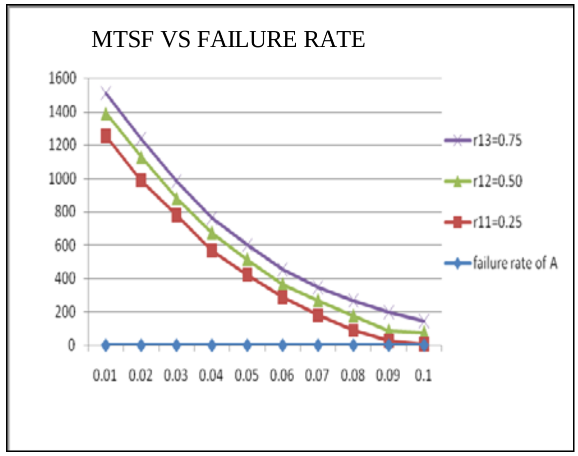


FIGURE-1

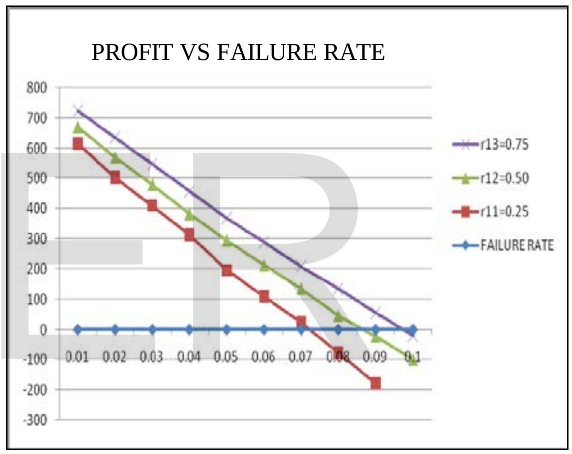


FIGURE-2

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